جـامـعـة بنهـــا

كلية الحاسبات والذكاء الإصطناعي

SC311
Modeling and Simulation

Lecture 03

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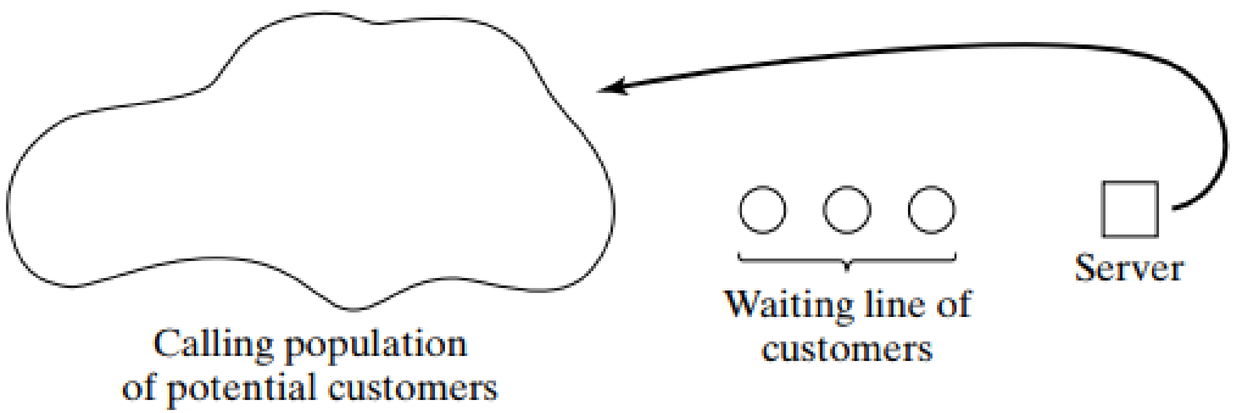
Chapter 3: Queueing Simulation

- Introduction.
- Description of Queuing System.
- Simulating a Single-Server Queue.
  - ✓ Example.
- Simulating a Queue with Multi-Servers.
  - ✓ Example.

Introduction (1/3)

Queueing Model:

- Simulation is often used in the analysis of queueing models. In a simple but typical queueing model, customers arrive from time to time and join a queue (waiting line), are eventually served, and finally leave the system.



Introduction (2/3)

- Queueing models, whether solved mathematically or analyzed through simulation, provide the analyst with a powerful tool for designing and evaluating the performance of queueing systems.
- Typical measures of system performance include server utilization (percentage of time a server is busy), length of waiting lines, and delays of customers.

Introduction (3/3)

- Quite often, when designing to improve a queueing system, the analyst (or decision maker) is involved in tradeoffs between server utilization and customer satisfaction in terms of line lengths and delays.

Description of Queuing Sys (1/8)

- The key elements of a queueing system are the ***customers*** and ***servers***.
- The term customer can refer to people, machines, trucks, mechanics, patients, airplanes, e-mail, cases, or orders—anything that arrives at a facility and requires service.
- The term server might refer to receptionists, repair personnel, mechanics, medical personnel, runways at an airport, automatic packers, order pickers, or CPUs in a computer—any resource (person, machine, etc.) that provides the requested service.

Description of Queuing Sys (2/8)

Queueing System:

1. The Calling Population.
2. System Capacity.
3. The Arrival Process.
4. Queue Behavior and Queue Discipline.
5. Service Times and the Service Mechanism.

Description of Queuing Sys (3/8)

1. The Calling Population (1/3):

- Also called: Input source (source of customers).
- The population of potential customers, referred to as the *calling population*, may be assumed to be finite or infinite.

Description of Queuing Sys (3/8)

1. The Calling Population (2/3):

- Infinite size:
  - Simulation assumptions: If a unit leaves the calling population and joins the waiting line or enters service, there is ***no change*** in the arrival rate of other units that may need service.
  - Most simulated models.
- Finite size:
  - Less common and complicates the simulation.

Description of Queuing Sys (3/8)

1. The Calling Population (3/3):

- The main difference between finite and infinite population models is how the arrival rate is defined. In an infinite population model, the arrival rate (i.e., the average number of arrivals per unit of time) is not affected by the number of customers who have left the calling population and joined the queueing system.

Description of Queuing Sys (4/8)

2. System Capacity:

- Is the maximum number of units that can be accommodated in the system.
- In many queueing systems, there is a limit to the number of customers that may be in the waiting line or system.
- It’s unlimited capacity if the system can accommodate any number of units in the waiting line.

Description of Queuing Sys (5/8)

3. The Arrival Process:

- The arrival process for infinite-population models is usually characterized in terms of interarrival times of successive customers. Arrivals may occur at scheduled times or at random times. When at random times, the interarrival times are usually characterized by a probability distribution.

Description of Queuing Sys (6/8)

4. Queue Behavior and Queue Discipline (1/2):

- *Queue behavior* refers to the actions of customers while in a queue waiting for service to begin. In some situations, there is a possibility that incoming customers will balk (leave when they see that the line is too long), renege (leave after being in the line when they see that the line is moving too slowly), or jockey (move from one line to another if they think they have chosen a slow line).

Description of Queuing Sys (6/8)

4. Queue Behavior and Queue Discipline (2/2):

- *Queue discipline* refers to the logical ordering of customers in a queue and determines which customer will be chosen for service when a server becomes free.

- Common queue disciplines include first-in-first-out (FIFO), last-in-first-out (LIFO), service in random order (SIRO), shortest processing time first (SPT), and service according to priority (PR).

Description of Queuing Sys (7/8)

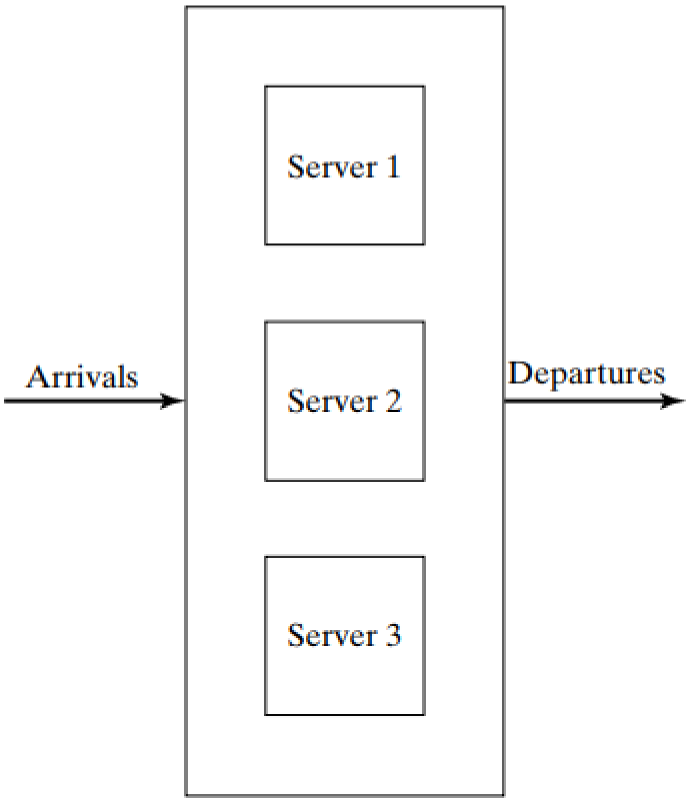
5. Service Times and the Service Mechanism (1/3):

- The service times of successive arrivals are denoted by $S_1, S_2, S_3, \ldots$. They may be constant or of random duration. In the latter case, $\{S_1, S_2, S_3, \ldots\}$ is usually characterized as a sequence of independent and identically distributed random variables.
- In addition, in some systems, service times depend upon the time of day or upon the length of the waiting line.

Description of Queuing Sys (7/8)

5. Service Times and the Service Mechanism (2/3):

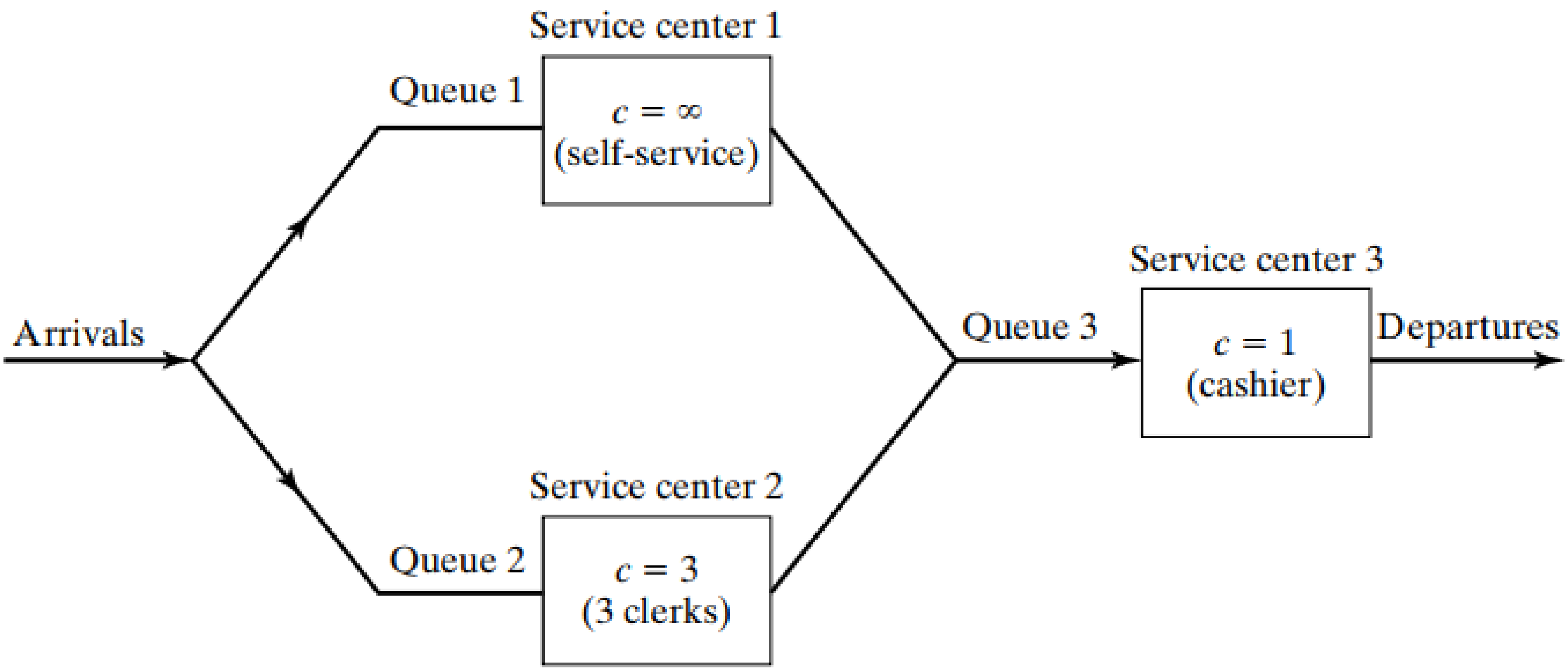
- A queueing system consists of a number of service centers and interconnecting queues. Each service center consists of some number of servers, c, working in parallel; that is, upon getting to the head of the line, a customer takes the first available server.



$c = 3$

Description of Queuing Sys (7/8)

5. Service Times and the Service Mechanism (3/3):



Description of Queuing Sys (7/8)

A notational system for parallel server systems: ***A***/***B***/***c***/***N***/***K***. These letters represent the following system characteristics:

A represents the interarrival-time distribution.

B represents the service-time distribution.

c represents the number of parallel servers.

N represents the system capacity.

K represents the size of the calling population.

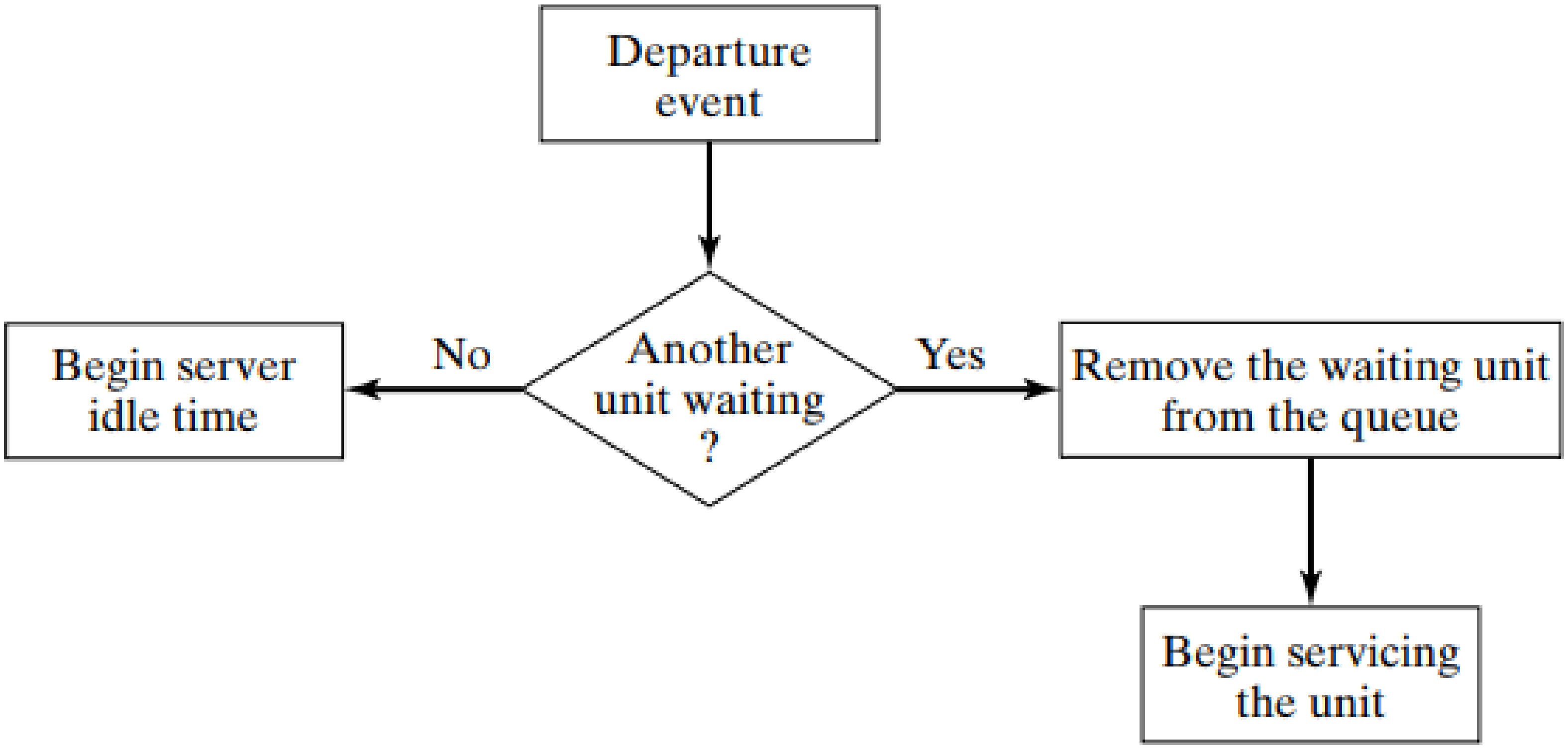
Description of Queuing Sys (8/8)

Queuing System State (1/3):

- The quantities (variables) that collectively completely describe the system at any given instant from the viewpoint of the study objectives:
  - **Number of units in the system:** How many units are currently in the queue or being processed by the server.
  - **Status of server (idle, busy):** Is the server(s) busy with processing some units or idle waiting for a job.

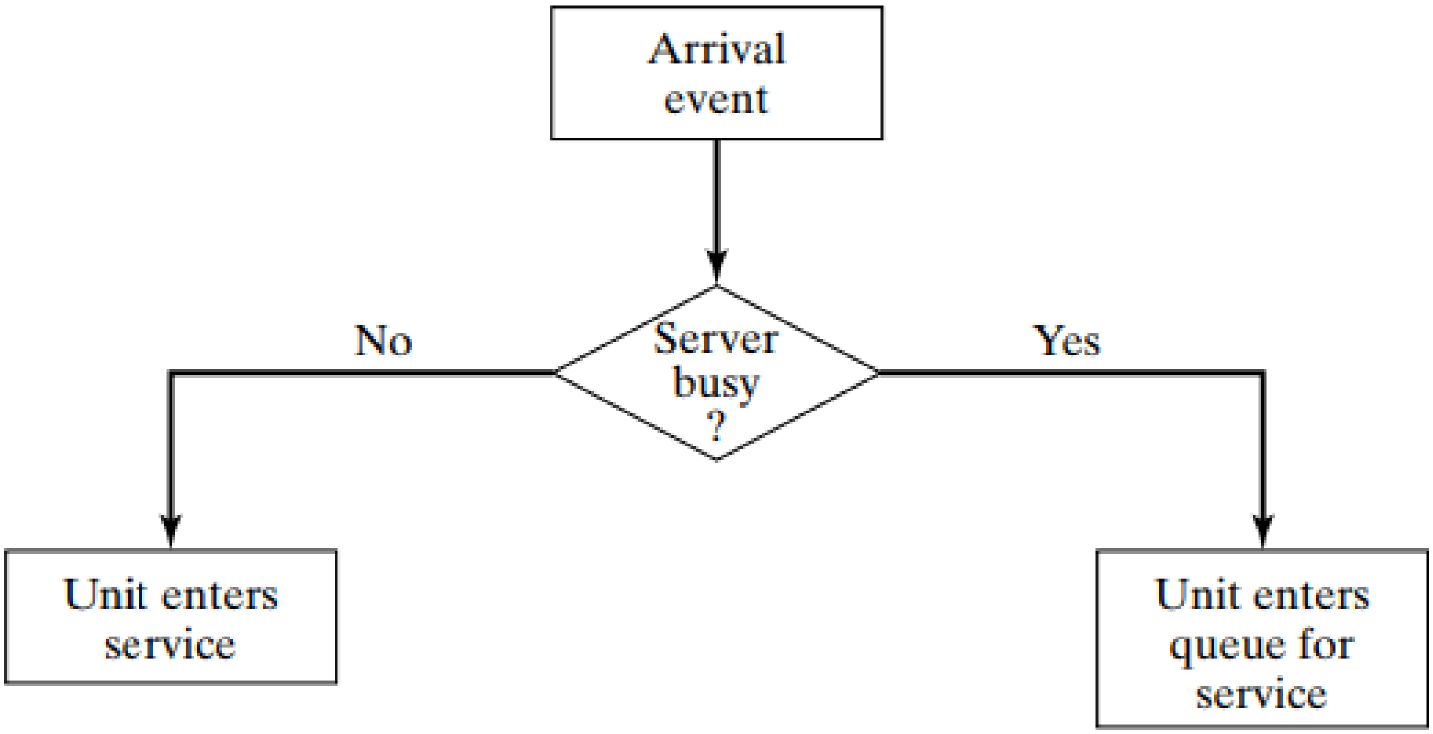
Description of Queuing Sys (8/8)

Queuing System State (2/3):



Description of Queuing Sys (8/8)

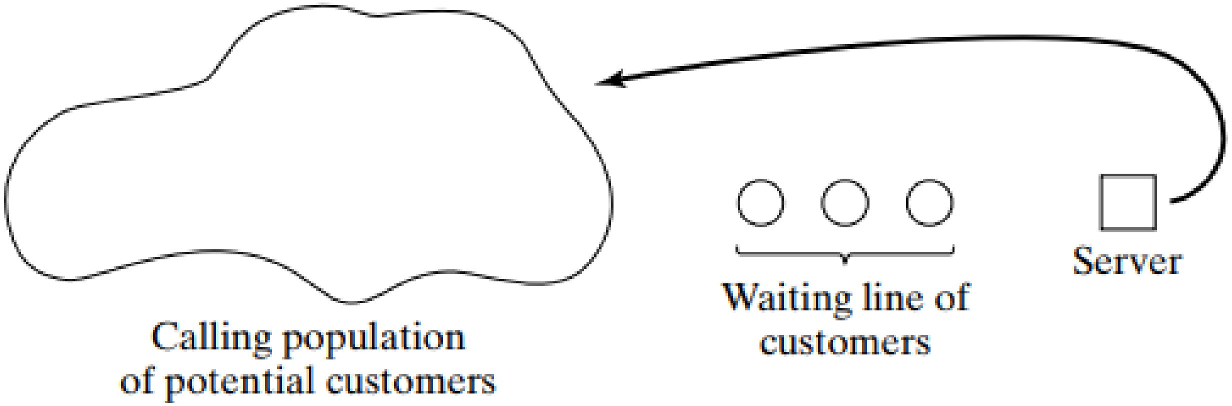
Queuing System State (3/3):



Sim. a Single-Server Queue (1/14)

Simulation Table (1/3):

- The following table was designed as a simulation table specifically for a single-channel queue that serves customers on a first-in–first-out (FIFO) basis.



Sim. a Single-Server Queue (1/14)

Simulation Table (2/3):

The interarrival and service times are taken from distributions!

Customer	Interarrival Time	Arrival Time on Clock	Service Time
1	-	0	2
2	2	2	1
3	4	6	3
4	1	7	2
5	2	9	1
6	6	15	4

The simulation run is build by meshing clock, arrival, and service times!

Customer Number	Arrival Time [Clock]	Time Service Begins [Clock]	Service Time [Duration]	Time Service Ends [Clock]
1	0	0	2	2
2	2	2	1	3
3	6	6	3	9
4	7	9	2	11
5	9	11	1	12
6	15	15	4	19

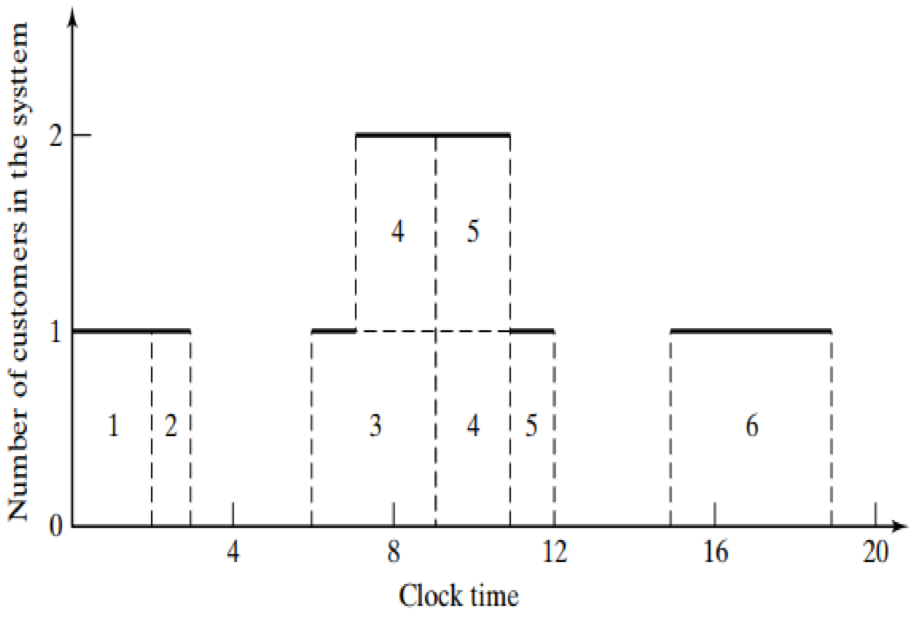
Sim. a Single-Server Queue (1/14)

Simulation Table (3/3):

Chronological ordering of events

Clock Time	Customer Number	Event Type	Number of customers
0	1	Arrival	1
2	1	Departure	0
2	2	Arrival	1
3	2	Departure	0
6	3	Arrival	1
7	4	Arrival	2
9	3	Departure	1
9	5	Arrival	2
11	4	Departure	1
12	5	Departure	0
15	6	Arrival	1
19	6	Departure	0

Interesting observations
- Customer 1 is in the system at time 0
- Sometimes, there are no customers
- Sometimes, there are two customers
- Several events may occur at the same time



Sim. a Single-Server Queue (2/14)

Example1

The Grocery Checkout, a Single-Server Queue:

Sim. a Single-Server Queue (3/14)

- A small grocery store has one checkout counter. Customers arrive at the checkout counter at random times that range from 1 to 8 minutes apart. We assume that interarrival times are integer-valued with each of the 8 values having equal probability; this is a discrete uniform distribution.

Interarrival Time [minute]
1
2
3
4
5
6
7
8

Sim. a Single-Server Queue (3/14)

- A small grocery store has one checkout counter. Customers arrive at the checkout counter at random times that range from 1 to 8 minutes apart. We assume that interarrival times are integer-valued with each of the 8 values having equal probability; this is a discrete uniform distribution.

Interarrival Time [minute]	Probability
1	0.125
2	0.125
3	0.125
4	0.125
5	0.125
6	0.125
7	0.125
8	0.125

Sim. a Single-Server Queue (3/14)

- A small grocery store has one checkout counter. Customers arrive at the checkout counter at random times that range from 1 to 8 minutes apart. We assume that interarrival times are integer-valued with each of the 8 values having equal probability; this is a discrete uniform distribution.

Interarrival Time [minute]	Probability	Cumulative Probability
1	0.125	0.125
2	0.125	0.250
3	0.125	0.375
4	0.125	0.500
5	0.125	0.625
6	0.125	0.750
7	0.125	0.875
8	0.125	1.000

Sim. a Single-Server Queue (4/14)

Distribution of Time Between Arrivals

Time between Arrivals (Minutes)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
1	0.125	0.125	001–125
2	0.125	0.250	126–250
3	0.125	0.375	251–375
4	0.125	0.500	376–500
5	0.125	0.625	501–625
6	0.125	0.750	626–750
7	0.125	0.875	751–875
8	0.125	1.000	876–000

Sim. a Single-Server Queue (5/14)

Time-Between-Arrivals Determination

Customer	*Random Digits*	*Time between Arrivals (Minutes)*	*Customer*	*Random Digits*	*Time between Arrivals (Minutes)*
1	–	–	11	109	1
2	913	8	12	093	1
3	727	6	13	607	5
4	015	1	14	738	6
5	948	8	15	359	3
6	309	3	16	888	8
7	922	8	17	106	1
8	753	7	18	212	2
9	235	2	19	493	4
10	302	3	20	535	5

Sim. a Single-Server Queue (6/14)

- The service times vary from 1 to 6 minutes (also integer-valued), with the probabilities shown in the following table.

Service Time [minute]	Probability	Cumulative Probability
1	0.10	0.10
2	0.20	0.30
3	0.30	0.60
4	0.25	0.85
5	0.10	0.95
6	0.05	1.00

Sim. a Single-Server Queue (7/14)

Service Time Distribution

Service Time (Minutes)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
1	0.10	0.10	01–10
2	0.20	0.30	11–30
3	0.30	0.60	31–60
4	0.25	0.85	61–85
5	0.10	0.95	86–95
6	0.05	1.00	96–00

Sim. a Single-Server Queue (8/14)

Service Times Generated

Customer	*Random Digits*	*Service Time (Minutes)*	*Customer*	*Random Digits*	*Service Time (Minutes)*
1	84	4	11	32	3
2	10	1	12	94	5
3	74	4	13	79	4
4	53	3	14	05	1
5	17	2	15	79	5
6	79	4	16	84	4
7	91	5	17	52	3
8	67	4	18	55	3
9	89	5	19	30	2
10	38	3	20	50	3

Sim. a Single-Server Queue (9/14)

Simulation Table for Queueing Problem (20 customers)

Simulation System						Performance Measure		
Customer	Interarrival Time [Minutes]	Arrival Time [Clock]	Service Time [Minutes]	Time Service Begins [Clock]	Time Service Ends [Clock]	Waiting Time in Queue [Minutes]	Time Customer in System [Minutes]	Idle Time of Server [Minutes]
1	—	0	4	0				
2	8		1					
3	6		4					
4	1		3					
5	8		2					
6	3		4					
7	8		5					
8	7		4					
9	2		5					
10	3		3					
11	1		3					
12	1		5					
13	5		4					
14	6		1					
15	3		5					
16	8		4					
17	1		3					
18	2		3					
19	4		2					
20	5		3					
Total	82		68					

Sim. a Single-Server Queue (9/14)

Simulation Table for Queueing Problem (20 customers)

Simulation System | Performance Measure

Customer	Interarrival Time [Minutes]	Arrival Time [Clock]	Service Time [Minutes]	Time Service Begins [Clock]	Time Service Ends [Clock]	Waiting Time in Queue [Minutes]	Time Customer in System [Minutes]	Idle Time of Server [Minutes]
1	—	0	4	0	4	0	4	0
2	8	8	1					
3	6		4					
4	1		3					
5	8		2					
6	3		4					
7	8		5					
8	7		4					
9	2		5					
10	3		3					
11	1		3					
12	1		5					
13	5		4					
14	6		1					
15	3		5					
16	8		4					
17	1		3					
18	2		3					
19	4		2					
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1	—	0	4	0	4	0	4	0
2	8	8	1	8	9	0	1	4
3	6		4					
4	1		3					
5	8		2					
6	3		4					
7	8		5					
8	7		4					
9	2		5					
10	3		3					
11	1		3					
12	1		5					
13	5		4					
14	6		1					
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1	—	0	4	0	4	0	4	0
2	8	8	1	8	9	0	1	4
3	6	14	4	14	18	0	4	5
4	1		3					
5	8		2					
6	3		4					
7	8		5					
8	7		4					
9	2		5					
10	3		3					
11	1		3					
12	1		5					
13	5		4					
14	6		1					
15	3		5					
16	8		4					
17	1		3					
18	2		3					
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Simulation Table for Queueing Problem (20 customers)

Customer	Interarrival Time [Minutes]	Arrival Time [Clock]	Service Time [Minutes]	Time Service Begins [Clock]	Time Service Ends [Clock]	Waiting Time in Queue [Minutes]	Time Customer in System [Minutes]	Idle Time of Server [Minutes]
	Simulation System					Performance Measure		
1	—	0	4	0	4	0	4	0
2	8	8	1	8	9	0	1	4
3	6	14	4	14	18	0	4	5
4	1	15	3	18	21	3	6	0
5	8		2					
6	3		4					
7	8		5					
8	7		4					
9	2		5					
10	3		3					
11	1		3					
12	1		5					
13	5		4					
14	6		1					
15	3		5					
16	8		4					
17	1		3					
18	2		3					
19	4		2					
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Sim. a Single-Server Queue (9/14)

Simulation Table for Queueing Problem (20 customers)

Simulation System						Performance Measure		
Customer	Interarrival Time [Minutes]	Arrival Time [Clock]	Service Time [Minutes]	Time Service Begins [Clock]	Time Service Ends [Clock]	Waiting Time in Queue [Minutes]	Time Customer in System [Minutes]	Idle Time of Server [Minutes]
1	—	0	4	0	4	0	4	0
2	8	8	1	8	9	0	1	4
3	6	14	4	14	18	0	4	5
4	1	15	3	18	21	3	6	0
5	8	23	2	23	25	0	2	2
6	3		4					
7	8		5					
8	7		4					
9	2		5					
10	3		3					
11	1		3					
12	1		5					
13	5		4					
14	6		1					
15	3		5					
16	8		4					
17	1		3					
18	2		3					
19	4		2					
20	5		3					
Total	82		68					

Sim. a Single-Server Queue (9/14)

Simulation Table for Queueing Problem (20 customers)

	Simulation System					Performance Measure		
Customer	Interarrival Time [Minutes]	Arrival Time [Clock]	Service Time [Minutes]	Time Service Begins [Clock]	Time Service Ends [Clock]	Waiting Time in Queue [Minutes]	Time Customer in System [Minutes]	Idle Time of Server [Minutes]
1	—	0	4	0	4	0	4	0
2	8	8	1	8	9	0	1	4
3	6	14	4	14	18	0	4	5
4	1	15	3	18	21	3	6	0
5	8	23	2	23	25	0	2	2
6	3	26	4	26	30	0	4	1
7	8		5					
8	7		4					
9	2		5					
10	3		3					
11	1		3					
12	1		5					
13	5		4					
14	6		1					
15	3		5					
16	8		4					
17	1		3					
18	2		3					
19	4		2					
20	5		3					
Total	82		68					

Sim. a Single-Server Queue (9/14)

Simulation Table for Queueing Problem (20 customers)

	Simulation System					Performance Measure		
Customer	Interarrival Time [Minutes]	Arrival Time [Clock]	Service Time [Minutes]	Time Service Begins [Clock]	Time Service Ends [Clock]	Waiting Time in Queue [Minutes]	Time Customer in System [Minutes]	Idle Time of Server [Minutes]
1	—	0	4	0	4	0	4	0
2	8	8	1	8	9	0	1	4
3	6	14	4	14	18	0	4	5
4	1	15	3	18	21	3	6	0
5	8	23	2	23	25	0	2	2
6	3	26	4	26	30	0	4	1
7	8	34	5	34	39	0	5	4
8	7	41	4	41	45	0	4	2
9	2	43	5	45	50	2	7	0
10	3	46	3	50	53	4	7	0
11	1	47	3	53	56	6	9	0
12	1	48	5	56	61	8	13	0
13	5	53	4	61	65	8	12	0
14	6	59	1	65	66	6	7	0
15	3	62	5	66	71	4	9	0
16	8	70	4	71	75	1	5	0
17	1	71	3	75	78	4	7	0
18	2	73	3	78	81	5	8	0
19	4	77	2	81	83	4	6	0
20	5	82	3	83	86	1	4	0
Total	82		68			56	124	18

Sim. a Single-Server Queue (9/14)

Simulation Table for Queueing Problem (20 customers)

Simulation System: Customer … Time Service Ends. Performance Measure: Waiting Time in Queue … Idle Time of Server.

Customer	Interarrival Time [Minutes]	Arrival Time [Clock]	Service Time [Minutes]	Time Service Begins [Clock]	Time Service Ends [Clock]	Waiting Time in Queue [Minutes]	Time Customer in System [Minutes]	Idle Time of Server [Minutes]
1	—	0	4	0	4	0	4	0
2	8	8	1	8	9	0	1	4
3	6	14	4	14	18	0	4	5
4	1	15	3	18	21	3	6	0
5	8	23	2	23	25	0	2	2
6	3	26	4	26	30	0	4	1
7	8	34	5	34	39	0	5	4
8	7	41	4	41	45	0	[illegible]	2
9	2	43	5	45	50	[illegible]	[illegible]	[illegible]
10	3	46	3	50	53	[illegible]	[illegible]	[illegible]
11	1	47	3	53	56	[illegible]	[illegible]	[illegible]
12	1	48	5	56	61	[illegible]	[illegible]	[illegible]
13	5	53	4	61	65	[illegible]	[illegible]	[illegible]
14	6	59	1	65	66	[illegible]	[illegible]	[illegible]
15	3	62	5	66	71	4	9	0
16	8	70	4	71	75	1	5	0
17	1	71	3	75	78	4	7	0
18	2	73	3	78	81	5	8	0
19	4	77	2	81	83	4	6	0
20	5	82	3	83	86	1	4	0
Total	82		68			56	124	18

Total Run Time of Simulation (86)

Sim. a Single-Server Queue (10/14)

Some of the findings from the simulation are as follows (1/7):

$$\text{average waiting time (minutes)} = \frac{\text{total time customers wait in queue (minutes)}}{\text{total numbers of customers}}$$

$$= \frac{56}{20} = 2.8 \text{ minutes}$$

Sim. a Single-Server Queue (10/14)

Some of the findings from the simulation are as follows (2/7):

$$\text{probability (wait)} = \frac{\text{number of customers who wait}}{\text{total number of customers}}$$

$$= \frac{13}{20} = 0.65$$

Sim. a Single-Server Queue (10/14)

Some of the findings from the simulation are as follows (3/7):

$$\text{probability of idle server} = \frac{\text{total idle time of server (minutes)}}{\text{total run time of simulation (minutes)}}$$

$$= \frac{18}{86} = 0.21$$

Sim. a Single-Server Queue (10/14)

Some of the findings from the simulation are as follows (4/7):

$$\text{average service time (minutes)} = \frac{\text{total service time (minutes)}}{\text{total number of customers}}$$

$$= \frac{68}{20} = 3.4 \text{ minutes}$$

This result can be compared with the expected service time by finding the mean of the service-time distribution using the equation:

$$E(S) = \sum_{s=0}^{\infty} s p(s)$$

$$= 1(0.10) + 2(0.20) + 3(0.30) + 4(0.25) + 5(0.10) + 6(0.05)$$

$$= 3.2 \text{ minutes}$$

Sim. a Single-Server Queue (10/14)

Some of the findings from the simulation are as follows (5/7):

$$\text{average time between arrivals (minutes)} = \frac{\text{sum of all times between arrivals (minutes)}}{\text{number of arrivals} - 1}$$

$$= \frac{82}{19} = 4.3 \text{ minutes}$$

The mean is given

$$E(A) = \frac{a+b}{2} = \frac{1+8}{2} = 4.5 \text{ minutes}$$

The expected time between arrivals is slightly higher than the average. However, as the simulation becomes longer, the average value of the time between arrivals will approach the theoretical mean, $E(A)$.

Sim. a Single-Server Queue (10/14)

Some of the findings from the simulation are as follows (6/7):

$$\text{Average waiting time of those who wait (minutes)} = \frac{\text{total time customers wait in queue (minutes)}}{\text{total number of customers who wait}}$$

$$= \frac{56}{13} = 4.3 \text{ minutes}$$

Sim. a Single-Server Queue (10/14)

Some of the findings from the simulation are as follows (7/7):

$$\text{average time customer spends in the system (minutes)} = \frac{\text{total time customers spend in the system (minutes)}}{\text{total number of customers}}$$

$$= \frac{124}{20} = 6.2 \text{ minutes}$$

The second way of computing this same result is to realize that the following relationship must hold:

$$= 2.8 + 3.4 = 6.2 \text{ minutes}$$

$$\begin{array}{c} \text{average time} \\ \text{customer spends} \\ \text{in the system} \\ \text{(minutes)} \end{array} = \begin{array}{c} \text{average time} \\ \text{customer spends} \\ \text{waiting in the} \\ \text{queue (minutes)} \end{array} + \begin{array}{c} \text{average time} \\ \text{customer spends} \\ \text{in service} \\ \text{(minutes)} \end{array}$$

Video Lectures

All Lectures: https://www.youtube.com/playlist?list=PLxIvc-MGOs6geFJmdvDOIN5zE89-Hq8Ij

Lecture #3: https://www.youtube.com/watch?v=ori_Zd-RsXs&list=PLxIvc-MGOs6geFJmdvDOIN5zE89-Hq8Ij&index=8

https://www.youtube.com/watch?v=Ie2oMOFbfw8&list=PLxIvc-MGOs6geFJmdvDOIN5zE89-Hq8Ij&index=9

https://www.youtube.com/watch?v=t2kJC3xjh54&list=PLxIvc-MGOs6geFJmdvDOIN5zE89-Hq8Ij&index=10

https://www.youtube.com/watch?v=XNvn303ZEqI&list=PLxIvc-MGOs6geFJmdvDOIN5zE89-Hq8Ij&index=11

https://www.youtube.com/watch?v=_crwcPS4GPE&list=PLxIvc-MGOs6geFJmdvDOIN5zE89-Hq8Ij&index=12

Thank You

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